



**Answer the following two questions:**

**Question 1**

(20 marks)

1. The accompanying two-way table records study the likes between the age of the mother when giving birth and her race. Define the following events:

A: {A mother is white}.

B: {A mother was a teenager when giving birth}.

Calculate:

a)  $P(A) \& P(B)$ , b)  $P(A \cup B)$ , c)  $P(A \cap B)$

2. A fair coin is tossed 10 times, and the up face is recorded after each toss. What is the probability of an event which observes at least one head?

3. Let a coin and a die be tossed.

a. Express explicitly the following events:

A: {a head and an even number appear}

B: {a prime number appears}

C: {a tail and an odd number appear}

b. For events A, B, and C, defined in part a, express explicitly the event that:

i. A or B occurs

ii. B and C occur

iii. Only B occurs

c. Which of the events A, B, C are mutually exclusive?

4. When a coin is tossed, heads are likely to appear twice as tails. Find  $P(T)$  and  $P(H)$ .

Mother's age	White	Black
$\leq 17$	2%	2%
18-19	3%	2%
20-28	41%	12%
$\geq 30$	33%	5%

**Question 2**

(20 marks)

1. When a die is tossed, the probability of the number appearing is proportional to the number itself.

Let A: {even number}, B: {prime number}, C: {odd number}.

a. Find the probability of each sample point of the sample space.

b. Find  $P(A)$ ,  $P(B)$ , and  $P(C)$ .

c. Find the probability that:

i. An even or prime number appears.

ii. An odd prime number appears.

iii. An even number but not prime number appears.

2. The students in a class are selected at random one after the other. Find the probability that the boys and girls in the class are alternates if the class consists of 4 boys and 3 girls; the first student must be a boy.

3. Box A contains 8 items of which 3 are defective, and box B contains 5 items of which 2 are defective. Two items, one from each box, are drawn at random.

a. What is the probability that both items are not defective?

b. What is the probability that one of the items is defective and the other is not?



- c. If one item is defective and the other is not, what is the probability that the defective item comes from box A?

### Question 3

(15 marks)

1. Let A be the event that a family has children of both genders, and let B be the event that a family has at most one boy.
- Show that A and B are independent events if the family has 3 children.
  - Show that A and B are dependent events if the family has 2 children.
2. The probability that a man will live more 10 years is  $1/4$ , and the probability that his wife will live more 10 years is  $1/3$ . Find the probability that:
- Both will be alive more 10 years.
  - At least one will live more 10 years.
  - Neither will live more 10 years.
  - Only the wife will live more 10 years.

3. Let  $f(x) = \begin{cases} x & , 0 \leq x \leq 1 \\ \frac{3-x}{4} & , 1 \leq x \leq 3 \\ 0 & , \text{elsewhere.} \end{cases}$

Prove that  $f(x)$  is a density function, then find the distribution function  $F(x)$ .

### Question 4

(15 marks)

1. Let  $x$  be a binomial random variable. Calculate the mean, variance and standard deviation for each of the following, where  $n$  is the number of samples and  $p$  is the probability of success:
- $n=80, p=0.2$
  - $n=70, p=0.9$
  - $n=1000, p=0.04$
2. Suppose the weight measurements  $W$  of 800 girls are normally distributed with a mean of 66 kg and a standard deviation of 5 kg. Find the number  $N$  of girls with weights:
- Between 65 and 70 kg.
  - Greater than or equal to 72 kg.  $\rightarrow 12$
3. If  $x$  is a continuous random variable with probability

$$P(x) = kx \quad 0 \leq x \leq 2, \text{ and zero elsewhere,}$$

find the cumulative distribution function, mean, variance, and standard deviation of  $x$ .

